

Using Language to Teach

For years, a slow weight gain for premature infants was accepted as normal. Recently, however, better understanding

of the special nutritional needs of these infants has resulted in a substantial increase in weight gains. In the same way, mathematics education research is giving us insights into helping young children learn substantially more mathematics.

Because international mathematics studies show that Asian children perform better than their age mates in the United States, I conducted a research project to study the effects of two essential elements of learning—language patterns and visualization—that are found in Japanese primary classrooms. I incorporated these elements into the mathematics instruction in a first-grade classroom in Minnesota.

The following discussion presents the rationale behind these two elements and describes the positive results of the instruction.

Language Patterns

Young children seek patterns as they learn about the world. The fact that patterns are predictable allows children to make great advances in their learning. For example, young children know that an *s* added to a word means *more than one* and that a *d* sound at the end of a word indicates past tense,

without having studied any English grammar.

Discovering the patterns for counting in English is difficult. The quantity ten has three names: *ten*, *-teen*, and *-ty*. The quantity three has another name, *thir-*, as in *third*, *thirteen*, and *thirty*; likewise, five's alias, *fif-*, appears in *fifth*, *fifteen*, and *fifty*. Also confusing are the numbers 11–19; the words *eleven* and *twelve* seem arbitrary, and for 13–19, the word order is reversed, with the ones stated before the tens. Besides blurring the pattern, these inconsistencies obscure the tens groupings.

Contrast the inconsistencies in English with the predictable patterns in most Asian languages, which follow the ancient Chinese method of number naming. The numbers 1–10 are single-syllable words. From 11–19, the names are *ten 1*, *ten 2*, *ten 3*, and so forth. The number 20 is named *2-ten*, and 21 is *2-ten 1*.

Counting to 100 in an Asian language requires learning the sequence 1–10 plus the word for 100, a total of just eleven words. In contrast, counting to 100 in English requires learning the words for 1–19 plus the decade names for 20 through 90, plus the word for 100, a total of twenty-eight words.

Korean is an interesting example for language study. The Korean language uses two distinct systems of number words—an everyday system with irregularities and the system described above, sometimes called the *academic system*. Song and Ginsburg (1988) studied how far Korean and American preschool children could count. They found that at four years of age, the average Korean child could count only to 8 in both systems; whereas the average American child could count to 22. At five years old, the average Korean child counted to 29 in the academic system and to 23 in the everyday system, and the American child, to 45. At six years old, after the Korean child learned the academic system in school, the average number reached in counting jumped to 91 in the academic system and to 61 in the everyday system. Meanwhile, at six years old, the average for American children was 72, showing

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and Visualization Place Value

approximately the same increase between four and five years old and between five and six years old. Korean children learn the counting sequence by recognizing counting patterns, whereas American children learn by rote memory.

Standard 13 in the NCTM's *Standards* (1989) recommends that children study patterns. The most important pattern is the base-ten number system. What better way to learn it than through a counting system that highlights that pattern?

Visualization

Visualization is an important part of the Japanese primary school curriculum. Mental images of quantities are necessary to work with these quantities mentally.



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To imagine or visualize a quantity, a person must be able to subitize it, that is, to recognize it immediately without counting. Subitizing has long been recognized as an important skill for developing number sense (Clements 1999).

Very young children can subitize small quantities. Wynn (1992) found that five-month-old babies can distinguish among one, two, and three objects. Researchers know that healthy babies will look longer at an object that is novel than one that is familiar. For example, Wynn showed a baby one teddy bear, which was then hidden behind a screen. Next, she showed the baby a second teddy bear and placed it behind the screen. A teddy bear was then added or removed from behind the screen when the baby's attention was focused elsewhere. When the screen was removed, Wynn measured the baby's viewing time. Babies showed significant differences in viewing times of correct and incorrect collections; it was found that they looked longer at incorrect collections. Strauss and Curtis (1981) found that half of twelve-month-old babies could distinguish up to four objects.

To help a young child associate the correct number with a small collection, we should refer to the whole collection by its number and avoid having children focus on individual objects through the counting ritual. In counting, the child loses the idea of the whole and assumes that we are tagging, or naming, each object. When we point and count four objects with a young child, then ask, "Show me the four," the child will often point to the fourth object rather than indicate the whole collection.

People cannot, however, recognize and visualize quantities of six to ten without some type of grouping. Try to imagine eight identical apples in a row without any grouping; the task is virtually impossible. Next imagine that five of those apples are red and that three are green. You can most likely visualize the eight. The Japanese use the five grouping for quantities of six to ten. The Romans constructed their numerals in groups of fives 2500 years ago; originally, they wrote IIII to represent 4, V for 5, VI for 6, and VIII for 9.

Manipulatives for Visualizing Number and Place Value

Manipulatives should enhance young children's abilities to visualize number through appropriate groupings. Sometimes dominoes are used for quick recognition, but dominoes have a serious limitation because they are not additive. For example, adding one dot to the five-dot pattern on a domino does not result in the six-dot pattern. A ten-frame, composed of a five-by-two grid with counters or dots placed in adjacent rectangles (see **fig. 1**), is a grouping that can be determined immediately without counting (Wirtz 1980). Ten-frames become cumbersome, however, for quantities greater than about thirty.

Japanese teachers emphasize visualization through their choice of manipulatives. The teachers use few manipulatives but require them to help children practice visualization. The manipulatives used in this study include fingers and tally sticks; the AL abacus, which is a special double-sided abacus with the beads grouped in fives through color (shown in **figs. 3–6**); and place-value cards.

Fingers and tally sticks

The most basic manipulative for children, especially young children, exists naturally on their hands—their fingers, and they are already grouped in fives! Use fingers for naming quantities, not counting. For quantities of one to five, children should use their left hands, to foster reading in the usual left-to-right sequence. For quantities of six to ten, children should use five on the left hand plus the amount over five on the right hand.

Tally sticks, or craft sticks, are an inexpensive tool for the next step in representing quantities. The children represent quantities of one to four by placing the sticks vertically about two centimeters apart. To represent five, children place a fifth stick horizontally across the four sticks; this action introduces the concept of grouping (see **fig. 2**). To emphasize the importance of ten, children are instructed to start a new row after each group of ten.

The AL abacus

For representing quantities to one hundred, the children use the AL abacus, which has two groups of five beads in contrasting colors strung on each of ten wires (see **fig. 3**). The children enter quantities by moving beads to the left and reading the quantities from left to right. The colors are reversed after five rows, helping the children subitize the number of tens. To develop number sense, children must operate on quantities in terms of tens and ones. The

quantity 7-ten 4 (74), for example, is simply seven rows of beads and four beads in the next row. Children can enter and visualize any quantity from one to one hundred without counting. The children can also construct hundreds by stacking abacuses. For example, to represent the quantity three hundred, three abacuses can be stacked.

This abacus configuration has several advantages over rods of varying lengths and colors that are often used to represent quantities:

- Young children frequently regard each rod, regardless of length, as a single unit.
- Eight percent of the population has some color deficiency and cannot see ten distinct colors.
- For rods representing the quantity five or greater, only the color, not the quantity, can be visualized.
- Combining two rods does not give the immediate sum; the child must either compare with a third rod or count. On the abacus, the result is seen immediately.
- When the sum is over 10, the rods do not reveal the tens structure. On the abacus, the resulting sum is seen immediately as a ten and ones.
- Quantities to one hundred can be subitized and visualized.
- The “counters” are self-contained, allowing more class time to be spent on concepts and less time on management of small pieces.

Place-value cards

To connect the physical representations with the pattern of written numbers, the children use place-value cards (shown in **fig. 4**), sometimes called *expanded-notation cards*. The set of place-value cards includes the numerals 1 through 9 printed on individual cards

FIGURE 1

Representing 7 with a ten-frame

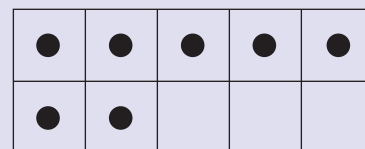
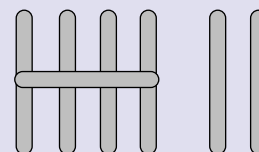


FIGURE 2

Representing 7 with tally sticks



of the same size; the numerals 10, 20, 30, . . . , 90 are printed on cards that are twice as wide; the numerals 100, 200, . . . , 900 are printed on cards that are three times as wide; and the numerals 1000, 2000, . . . , 9000 are printed on cards that are four times as wide.

The teacher introduces, for example, the 3-ten (30) card by pointing to the 3 and saying, “three,” then pointing to the 0 and saying, “ten.” Likewise, for 400, the teacher points to the 4 and says, “four,” then points to the two 0s in succession, saying “hundred.” To construct 37, the child places the 7 card on top of the 0 of the 30 card. Using these cards, a child learns to recognize the tens digit by the single digit following it, not by its placement in a particular column.

Using these place-value cards has some interesting advantages over using the column model:

- The child sees a number, such as 37, as 30 and 7, not merely as 3 joined to 7, thereby avoiding a common error. Reversals are all but impossible.
- The child reads the numbers in the normal left-to-right pattern, not backward, as in the column model, in which a child starts at the right and says, “ones, tens, hundreds.”
- The child can read the number 100 as “one hundred” or as “10-ten,” which indeed it is. Also, a number such as 1200 makes sense read as “12 hundred.”

Using these hands-on tools, children begin to visualize and construct for themselves the patterns of our number system. These skills enable them to understand computation and develop efficient strategies for learning the facts.

Visualization and Computation

In the United States, counting is considered the cornerstone of arithmetic; children engage in various counting strategies: counting all, counting on, and counting back. Japanese teachers have a different view of counting. Starting in first grade, students in Japan are discouraged from using one-by-one counting procedures. According to Hatano (1982), Japanese researchers found that mere practice in counting did not help children advance in the conservation tasks identified by Piaget. The Japanese Council of Mathematics Education states that children who are taught counting procedures have more difficulties in solving story problems, although they do well in computation (Hatano 1982, p. 215).

Counting, which is slow and unreliable until six years of age, creates other difficulties, as

FIGURE 3

Representations on the AL abacus

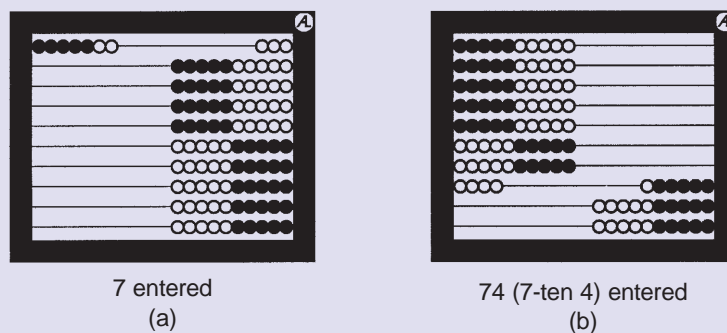
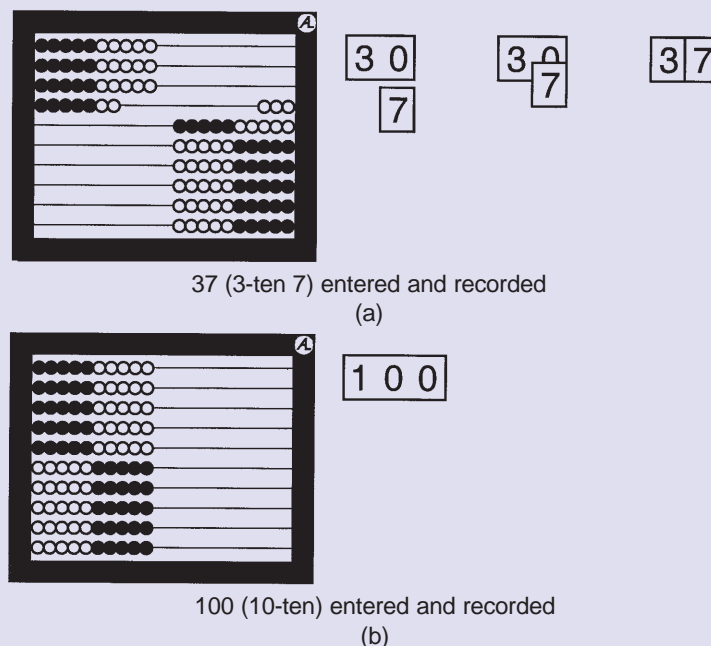


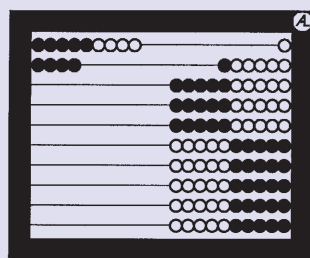
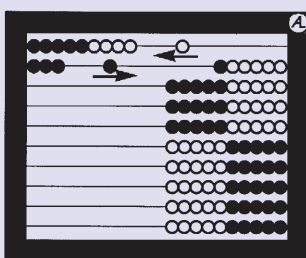
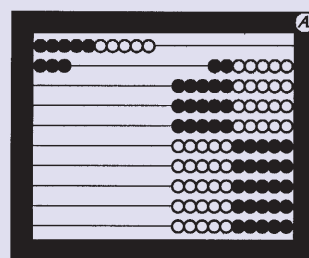
FIGURE 4

Representations with the abacus and place-value cards



well. Children who use counting to add and subtract develop a unitary concept of number. That is, they think of 14 as 14 ones, not as 1 ten and 4 ones. Such thinking interferes with their understanding of place-value concepts and often results in rote learning of algorithms. To understand that our number system is based on tens, children must experience the pattern of trading: 10 ones for 1 ten, 10 tens for 1 hundred, and 10 hundreds for 1 thousand.

Visualization strategies offer efficient techniques for learning the facts. In the “complete the ten” strategy, for example, to add $9 + 4$, the child enters 9 and 4 on the top two wires of the AL abacus; as the next step, he or she takes 1 from the 4 and combines it with the 9 to get 10 and 3, or 13 (see **fig. 5**). The child progresses from entering and rearranging the quantities physically to entering the quantities physically but rearranging them mentally, then to performing the entire

Using the “complete the ten” strategy to add $9 + 4$ Enter 9 and 4 on separate wires
(a)Remove 1 from the 4, and give it to the 9.
(b)The sum is $10 + 3$, or 13.
(c)

process mentally. This strategy also works for higher numbers; for example, this same technique could be used for $59 + 4$.

Another strategy that lends itself to visualization is the “2 fives” strategy, which works when both addends are 5 or more. For example, to add $8 + 6$, the child enters 8 and 6 on the top two wires of the abacus, as shown in **figure 6**. The 2 fives formed by the dark-colored beads make a ten, and the amounts over 5, which are 3 and 1, total 4, to yield a sum of 14.

A Classroom Study

During the 1994–1995 school year, I conducted a study in two first-grade classrooms, each with sixteen children, in a rural Minnesota community (Cotter 1996). The experimental class used lesson materials that I supplied, whereas the matched control class used a traditional workbook.

For the first three months of the school year, the experimental class used the “Asian” method of counting with a slight modification for the numbers in the teens. I had the children say, “1-ten 3” rather than “ten 3.” Parents had no complaints about this method. When introduced to 2-ten, one boy said that the number was twenty. The teacher told him that for now the class would call the number 2-ten, and the student was satisfied with that explanation.

The children learned the finger combinations to 10. They practiced both holding up the requisite number of fingers when asked and recognizing the quantity on the teacher’s hands. They also practiced laying out and recognizing tiles or counters, but always being asked to change color or some other attribute after five to facilitate subitizing and visualizing.

The children easily learned to enter quantities from one to ten on the abacus without counting. They added two quantities with a sum of less

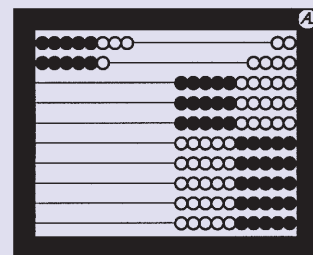
than 10 by entering the two quantities in tandem and reading the sum immediately.

Around the fourth week of first grade, the teacher introduced tens. The children practiced entering various tens up to 10-ten. They soon discovered that adding 2-ten and 2-ten followed the pattern of adding 2 and 2. They added two quantities, such as $8 + 6$, by entering 8 and 2 from the 6 to fill the first row, then entering the remaining 4 from the 6 in the second row. The sum was apparent.

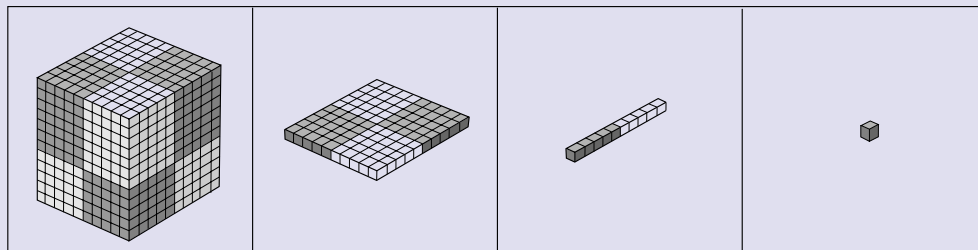
The children constructed quantities using the abacuses and the place-value cards. When the stacks of abacuses became cumbersome for large numbers, the children used four-centimeter-square cards with drawings of thousands, hundreds, tens, and ones (see **fig. 7**).

The children also used these base-ten cards for adding with trading. Each child in a group of three constructed a four-digit number with the cards. Then the group added its numbers by combining its cards and trading 10 of any denomination for 1 of the next higher denomination. They displayed their sums using place-value cards and recorded the sums on paper.

Along one edge of the reverse side of the AL abacus is a label that designates two wires each for the thousands, hundreds, tens, and ones; the

Using the “2 fives” strategy to add $8 + 6$ (the dark-colored beads equal 10, and the light-colored beads equal 4, giving a sum of 14)

Base-ten picture cards representing thousand, hundred, ten, and one



third and seventh wires are not used (see **fig. 8**). Placing the abacus so that the label is at the top, the children entered four-digit numbers by moving up the requisite number of beads on the vertical wires. Using two wires allows an ample number of beads for entering both addends before trading.

For example, to add $8 + 6$, the child enters 8 beads, followed by 6 beads (see **fig. 8**). The child can see the sum without counting, because the light-colored beads form a ten. Because no more than 9 can be recorded in any denomination, a trade is necessary. To trade, the child moves up 1 tens bead with the left hand while moving down 10 ones beads with the right hand.

The children enjoyed the bead-trading activity and benefited from the trading practice. They also added the numbers on cards labeled 1–10 by entering the numbers in the ones columns and trading when necessary. When the children mastered trading, they began entering and adding four-digit numbers. A few days later, they figured out for themselves how to perform addition on paper without the abacus.

During the fourth month of first grade, the children learned the traditional number names. First, the teacher explained that 4-ten has another name, *forty*, emphasizing that *ty* means *ten*. In the same way, she taught the numbers 60, 70, 80, and 90. For 30 and 50, she also used the words *third* and *fifth* to remind the students of the new names. *Twin-ten* helped them remember *twenty*.

Before introducing the teen names, the teacher played a word game in which the children reversed the syllables in such words as *sunset* and *bedroom*, changing them to *setsun* and *roombed*. Then the teacher told the students that *teen* means *ten*. Ten-4 becomes “teen four,” and reversing the syllables completes the transformation to “fourteen.” The children learned the names for 16–19 in the same way, then 13 and 15, which need the *thir* and *fif* modification.

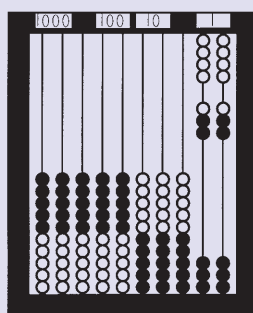
The teacher explained the word *eleven* by laying out eleven objects with ten in a row and one below it. People once referred to this quantity by noticing the one object left over: “a one left,” or

with the words reversed, “left one.” Eventually, this number came to be known as “eleven.” The word *twelve* was derived in the same way: “two left.”

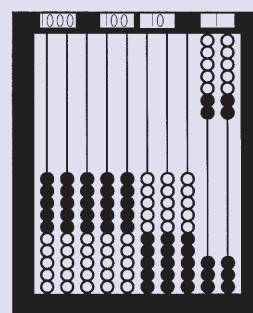
Results

After the experimental lessons, the teacher remarked that the children had advanced much farther than she had ever expected. She thought that the children who had learning difficulties learned much more than they would have achieved in a traditional program. Zachery, a first grader with learning difficulties, was able to add nines mentally

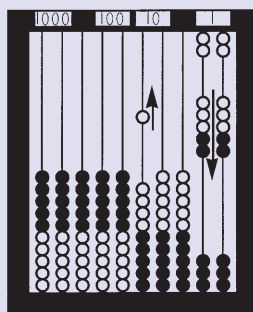
Trading on the reverse side of the AL abacus



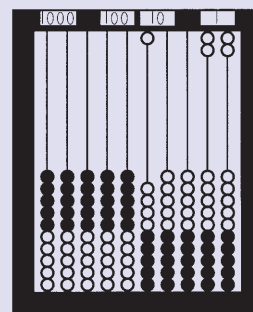
Enter 8 ones and 6 ones using two wires.
(a)



Show the sum as 14 (the light-colored beads form a 10).
(b)



Trade 10 ones for 1 ten.
(c)



Show the sum as 14 after the trade.
(d)

**In the United States,
children spend years
learning to package
numbers efficiently**

using the “complete the ten” strategy.

At the end of the eighth month, I interviewed all the children in both classes. Using a procedure from Miura and others (1993), I gave the first-grade children tens and ones from base-ten blocks and showed them that the ten was equal to 10 ones. Then I asked

the children to use the blocks to make 11, 13, 28, 30, and 42. In the interviews, 21 percent of the students in the experimental class and 44 percent of the students in the control class made their constructions without any tens blocks. Zachery made all five of his constructions with tens blocks and nine or fewer ones.

During the interviews, I also asked the children to construct 48 with the tens and ones blocks, then to

subtract 14. Note that no “borrowing” is involved. Children who had a unitary concept of number removed 14 ones, that is, 8 separate ones and 6 more from a ten. In the experimental class, 81 percent removed a ten and 4 ones, but only 33 percent of the control class did so.

Some other results of the interviews are as follows:

- In the experimental class, 94 percent knew the sum of $10 + 3$ and 88 percent knew $6 + 10$; in the control class, 47 percent knew $10 + 3$ and 33 percent knew $6 + 10$.
- When asked to circle the tens place in the number 3924, 44 percent of the experimental class and 7 percent of the control class did so correctly.
- When asked to compute $85 - 70$ mentally, 31 percent of the experimental class did so correctly; none of the students in the control class did so.
- Forty percent of the students in the control class wrote 512 for the sum of $38 + 24$ or 812 for $57 + 35$; none of the experimental class did so, even without using the abacus.

Concluding Comments

In the United States, we give our children numbers in bulk and they spend years learning to package the numbers efficiently. Asians give their children prepackaged numbers. Teaching children to count initially with consistent counting words follows good teaching practice; introducing exceptions should occur only after the students understand the general rule.

Some teachers and parents are concerned that the

abacus may become a crutch. The best answer to this issue was given by five-year-old Stan from my class after I asked him how he knew that $11 + 6$ is 17. He replied by saying, “because I’ve got the abacus in my mind.” Once the children visualize a concept, they do not use the abacus. A child who can visualize quantities and understand the patterns in the number system has developed good number sense and will search for more patterns, a quest that leads to an increase in abstract thought.

Young children are capable of adding and subtracting and performing other mathematical tasks before they develop accurate counting skills. Counting need not be the basis of arithmetic. These Minnesota children and subsequent classes have developed an understanding of our number system and efficient strategies for learning the facts. Language patterns and visualization are two components of learning that help young children construct mathematical knowledge.

Bibliography

- Activities for Learning. AL Abacus, n.d. Manipulative. (Available from Activities for Learning, 129 S.E. Second St., Linton, ND 58552.)
- Clements, Douglas H. “Subitizing: What Is It? Why Teach It?” *Teaching Children Mathematics* 5 (March 1999): 400–404.
- Cotter, Joan A. *Activities for the AL Abacus: A Hands-on Approach to Arithmetic*. 2nd ed. Hutchinson, Minn.: Activities for Learning, 1988.
- . “Constructing a Multidigit Concept of Numbers: A Teaching Experiment in the First Grade.” Ph.D. diss., University of Minnesota, 1996. Abstract in *Dissertation Abstracts International* 9626354, DAI-A 57/04, p. 1465.
- Hatano, Giyoo. “Learning to Add and Subtract: A Japanese Perspective.” In *Addition and Subtraction: A Cognitive Perspective*, edited by Thomas P. Carpenter, James M. Moser, and Thomas A. Romberg, pp. 211–23. Hillsdale, N.J.: Lawrence Erlbaum Associates, 1982.
- Miura, Irene T., Yukari Okamoto, Chungsoon C. Kim, Marcia Steere, and Michel Fayol. “First Graders’ Cognitive Representation of Number and Understanding of Place Value: Cross-National Comparisons—France, Japan, Korea, Sweden, and the United States.” *Journal of Education Psychology* 85 (January 1993): 24–30.
- National Council of Teachers of Mathematics (NCTM). *Curriculum and Evaluation Standards for School Mathematics*. Reston, Va.: NCTM, 1989.
- Piaget, Jean. *The Child’s Conception of Number*. New York: W. W. Norton & Co., 1966.
- Song, Myung J., and Herbert P. Ginsburg. “The Effect of the Korean Number System on Young Children’s Counting: A Natural Experiment in Numerical Bilingualism.” *International Journal of Psychology* 23 (1988): 319–32.
- Strauss, Mark S., and Lynne E. Curtis. “Infant Perception of Numerosity.” *Child Development* 52 (December 1981): 1146–52.
- Wirtz, Robert. *New Beginnings*. Monterey, Calif.: Curriculum Development Associates, 1980.
- Wynn, Karen. “Addition and Subtraction by Human Infants.” *Nature* 358 (27 August 1992): 749–50. ▲